

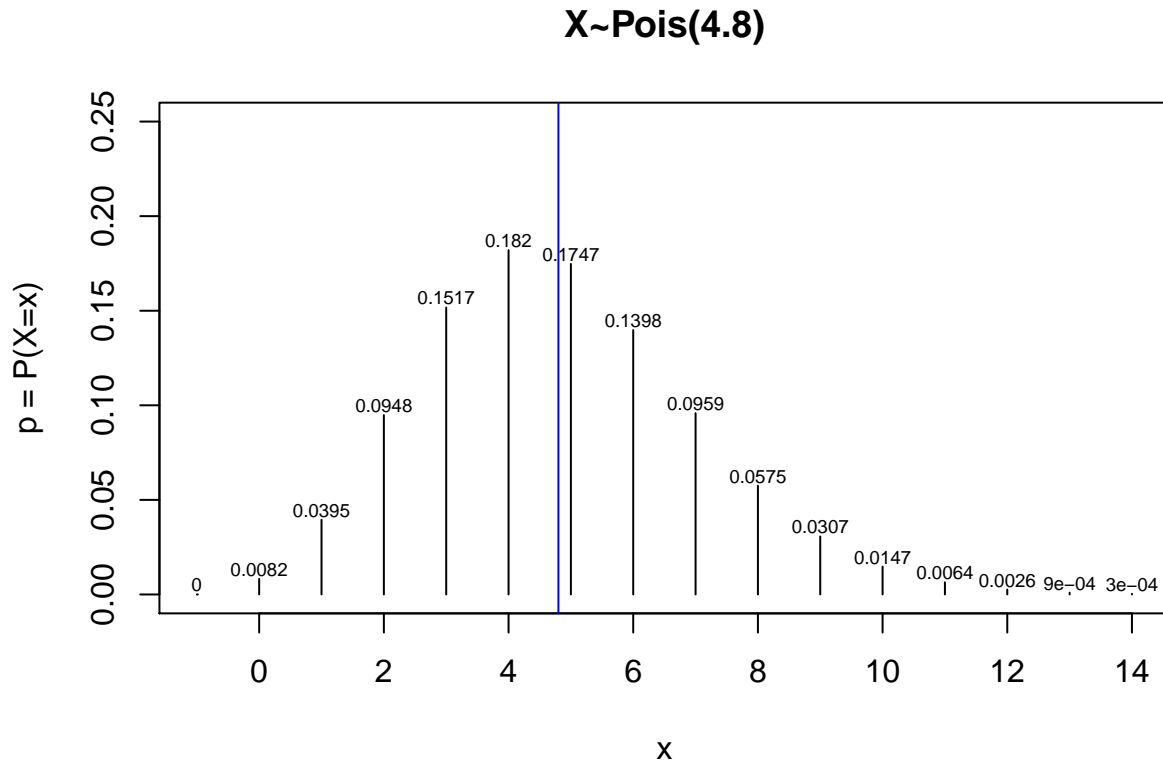
# Poisson pmf and CDF

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## pmf

We can use the base plot functions in R to create a plot of the pmf for a Poisson random variable  $X$  with  $\lambda = 4.8$  — i.e.  $X \sim \text{Pois}(4.8)$ .

```
lambda <- 4.8
x <- -1:14
pmf <- dpois(x, lambda)
plot(x, pmf, type="h", xlab="x", ylab="p = P(X=x)", main="X~Pois(4.8)", ylim=c(0, 0.25), xaxt="n")
axis(1, at=seq(0,14,by=2))
text(x, pmf+0.005, round(pmf, digits=4), cex=0.6)
abline(v=lambda, col="blue")
```

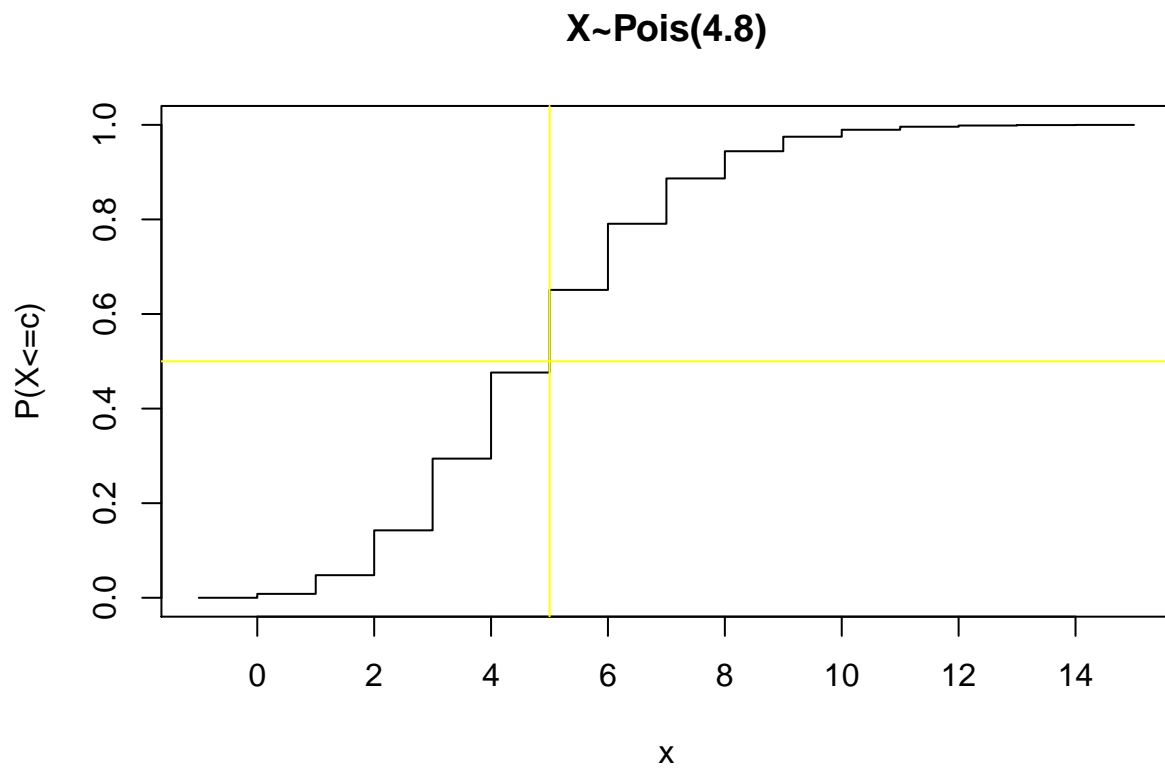


## CDF

The CDF may be plotted analogously.

```
x <- -1:15
cdf <- ppois(x, lambda)
plot(x, cdf, type="s", xlab="x", ylab="P(X<=c)", main="X~Pois(4.8)", xaxt="n")
axis(1, at=seq(0,14,by=2))
abline(h=0.5, col="yellow")
```

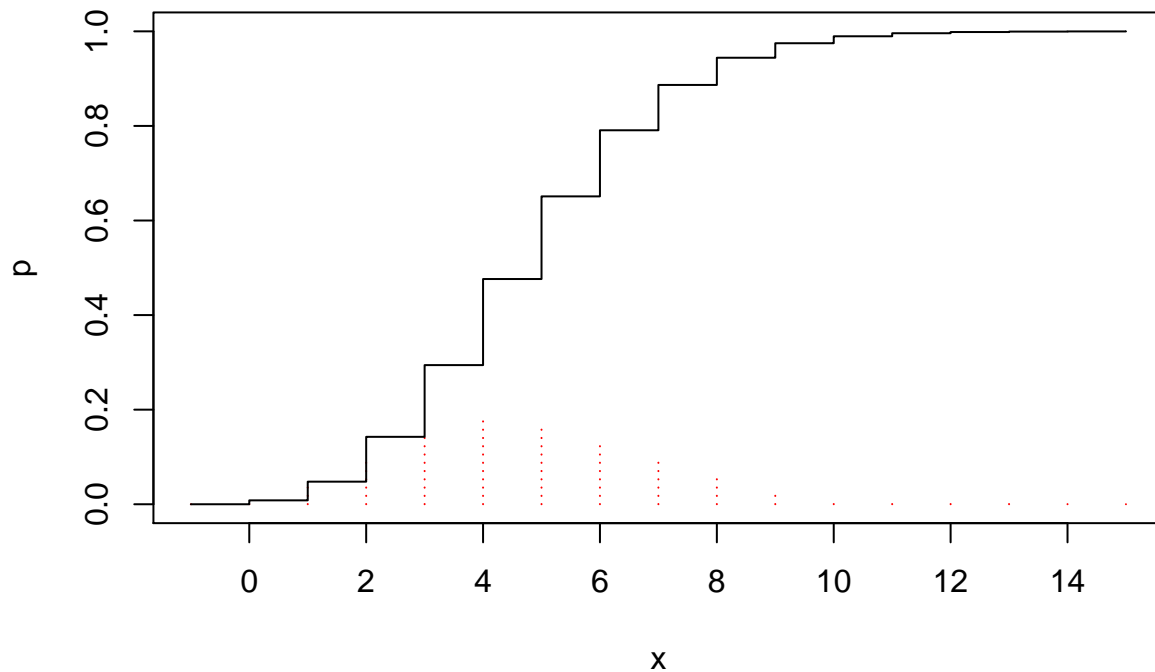
```
abline(v=qpois(0.5,lambda), col="yellow")
```



Just for fun we can overlay the two.

```
pmf <- dpois(x, lambda)
plot(x, pmf, type="h", xlab="x", ylab="p", main="X~Pois(4.8)", ylim=c(0, 1), xaxt="n", col="red", lty=1)
lines(x, cdf, type="s", xlab="x", ylab="P(X<=c)", main="X~Pois(4.8)", xaxt="n")
axis(1,at=seq(0,14,by=2))
```

## X~Pois(4.8)



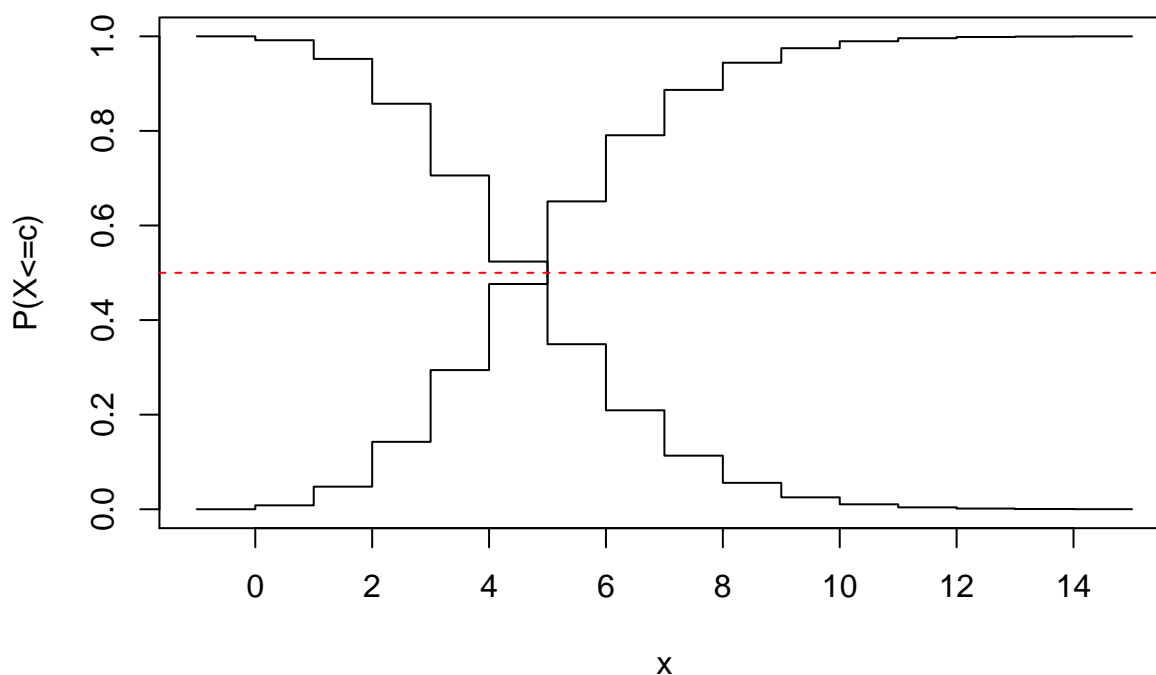
Now look for the median.

```
data.frame(x,pmf,cdf,invcdf=1-cdf)
```

```
##      x      pmf      cdf      invcdf
## 1  -1 0.0000000000 0.000000000 1.000000e+00
## 2   0 0.0082297470 0.008229747 9.917703e-01
## 3   1 0.0395027858 0.047732533 9.522675e-01
## 4   2 0.0948066860 0.142539219 8.574608e-01
## 5   3 0.1516906976 0.294229916 7.057701e-01
## 6   4 0.1820288371 0.476258754 5.237412e-01
## 7   5 0.1747476836 0.651006437 3.489936e-01
## 8   6 0.1397981469 0.790804584 2.091954e-01
## 9   7 0.0958615865 0.886666171 1.133338e-01
## 10  8 0.0575169519 0.944183123 5.581688e-02
## 11  9 0.0306757077 0.974858830 2.514117e-02
## 12 10 0.0147243397 0.989583170 1.041683e-02
## 13 11 0.0064251664 0.996008336 3.991664e-03
## 14 12 0.0025700666 0.998578403 1.421597e-03
## 15 13 0.0009489477 0.999527350 4.726495e-04
## 16 14 0.0003253535 0.999852704 1.472960e-04
## 17 15 0.0001041131 0.999956817 4.318292e-05
```

```
plot(x,cdf,type="s", xlab="x", ylab="P(X<=c)", main="X~B(13, 0.6)", xaxt="n")
lines(x,1-cdf,type="s", xlab="x", xaxt="n")
axis(1,at=seq(0,14,by=2))
abline(h=0.5, col="red", lty=2)
```

## $X \sim B(13, 0.6)$



## Poisson Approximation to the Binomial

We now look at how well the Poisson approximates the binomial.

```
n <- 100
p <- 0.01
x <- 0:3
pois.pmf <- dpois(x, n*p)
pois.cdf <- ppois(x, n*p)
binom.pmf <- dbinom(x, n, p)
binom.cdf <- pbinom(x, n, p)
data.frame(pois.pmf, pois.cdf, binom.pmf, binom.cdf)
```

```
##      pois.pmf  pois.cdf  binom.pmf  binom.cdf
## 1 0.36787944 0.3678794 0.36603234 0.3660323
## 2 0.36787944 0.7357589 0.36972964 0.7357620
## 3 0.18393972 0.9196986 0.18486482 0.9206268
## 4 0.06131324 0.9810118 0.06099917 0.9816260
```

## Typhoid Example

We let R plot the values that we computed in class.

```
x <- 0:25
pmf.24 <- dpois(x, 2.4)
pmf.48 <- dpois(x, 4.8)
pmf.96 <- dpois(x, 9.6)
```

```
plot(x, pmf.24, pch=1, col=1, xlab="x", ylab="p")
points(x, pmf.48, pch=2, col=2)
points(x, pmf.96, pch=3, col=3)
abline(v = c(2.4, 4.8, 9.6), col=1:3, lty=2)
```

